Negative energy state of electron

or

Theary of Positron or Hole Theory :-

Explaination of Pair - production and Pair - Annhilation :-

The Dirac eqⁿ in electromagnetic field is given by :-

$$[E - e\phi - \vec{\alpha}.(cp - eA) - \beta m c^2]\psi = 0 \quad \text{---(1)}$$

The above eqⁿ may also be written as four equivalent eqⁿ, two corresponding to positive energy and the remaining two corresponding to negative energy state of the particle. According to classical mechanics negative energy states can be excluded on physical grounds ; but according to quantum mechanics it is not so because energy changes take place discontinuously and the transition from positive

to negative energy states and vice-versa are possible.

We have eqⁿ (1) as a matrix eqⁿ assuming all the elements of matrices representing $\alpha_x, \alpha_y, \alpha_z$ to be real and all those of matrix representing β to be purely imaginary or zero. Then we take complex conjugate of (1)

$E = i\hbar \frac{\partial}{\partial t}$ and $p = -i\hbar\nabla$,

we get

$$[(-E - e\phi) - \vec{\alpha}.(-cp - eA) + \beta mc^2]\psi^* = 0$$

or

$$[E + e\phi] - \vec{\alpha}[(c\vec{p} + e\vec{A}) - \beta mc^2]\psi^* = 0 \quad \text{---(2)}$$

Thus each solⁿ of the wave-eqⁿ (1) has for its complex conjugate a solution of (2). Further if the solⁿ ψ of (1) belongs to a negative value for $(E - e\phi)$ the corresponding solⁿ ψ^* of eqⁿ (2) will belong to a positive energy for $(E + e\phi)$. But the operator in eqⁿ (2) is just what one would get if one substitutes $-e$ for e in the operator of (1). It follows that each negative energy solⁿ of the wave eqⁿ is obtained from (1) by substitution of $-e$ for e. This latter solⁿ represents an e^- of charge $+e$ instead of $-ve$ as we had upto the present) moving

The soln of eqn (1) was given by Dirac to explain the negative energy of e^-.

According to Dirac, the energy becomes more negative & negative until the particle emits photon, radiation and it goes to $(-\infty)$ i.e; energy becomes decay - decaying and there is no end of it. This means that no one factor can prevent to decay this lower energy.

Dirac tried at first to find the negative energy soln and consider that this negative energy soln is due to proton. But both the positive and negative energy solutions have the same mass while, proton and e^- do not have the same mass. Thus, there was only one choice before Dirac to predict the anti-particle i.e; e^- and positron. This theory of Dirac was justified as the theory of positron obeys Pauli's